

WA8MMGTW3

Two-dimensional space and structures

Core + Challenge

Pythagoras' Theorem: Finding a Shorter Side — Marking Key

I can rearrange Pythagoras' theorem to find a shorter side and decide whether to add or subtract.

For tutors and parents. Award method marks for correct working even if the final answer is wrong.

Marks summary

Section A Fluency: 16 Section B Core: 22 Section C Reasoning: 9 Section D Challenge: 11 **Total: 58**

Fluency: 1 mark each; accept an equivalent unsimplified answer for half credit at tutor's discretion.

A Fluency *answers and working*

/ 16 marks

Q	Answer	Working
1	subtract	x is a shorter side
2	8 cm	$\sqrt{(100 - 36)} = \sqrt{64}$
3	12 cm	$\sqrt{(169 - 25)} = \sqrt{144}$
4	17 cm	hypotenuse: $\sqrt{(64 + 225)} = \sqrt{289}$
5	24 cm	$\sqrt{(625 - 49)} = \sqrt{576}$
6	64	$100 - 36$
7	64	$289 - 225$
8	5	$\sqrt{(169 - 144)} = \sqrt{25}$
9	15	$\sqrt{(625 - 400)} = \sqrt{225}$
10	40	$\sqrt{(1681 - 81)} = \sqrt{1600}$
11	2	$\sqrt{(6.25 - 2.25)} = \sqrt{4}$
12	11	$\sqrt{(3721 - 3600)} = \sqrt{121}$
13	7.1	$\sqrt{51} = 7.141... \approx 7.1$
14	10.9	$\sqrt{119} = 10.908... \approx 10.9$
15	7.4	$\sqrt{55} = 7.416... \approx 7.4$
16	not possible	the hypotenuse must be the longest side

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B

Core *answers, working and marks*

/ 22 marks

Q	Answer	Working / sample response	Marks
1	x = 24 cm	$x^2 = 26^2 - 10^2 = 676 - 100 = 576$ $x = \sqrt{576} = 24$ <i>1 mark: subtraction</i> <i>1 mark: 24 cm</i>	2
2	y ≈ 10.2 m	$y^2 = 15^2 - 11^2 = 225 - 121 = 104$ $y = \sqrt{104} = 10.198... \approx 10.2$ <i>1 mark: 104</i> <i>1 mark: 10.2 m</i>	2
3	p = 34 cm (hypotenuse); q = 16 cm (shorter side)	(a) p is opposite the right angle: $p^2 = 256 + 900 = 1156$, p = 34 (b) q is a shorter side: $q^2 = 400 - 144 = 256$, q = 16 <i>1 mark: correct decisions</i> <i>1 mark: p = 34</i> <i>1 mark: q = 16</i>	3
4	14 cm	$h^2 = 50^2 - 48^2 = 2500 - 2304 = 196$ $h = \sqrt{196} = 14$ <i>1 mark: 196</i> <i>1 mark: 14 cm</i>	2
5	6 cm	$a^2 = 56.25 - 20.25 = 36$ $a = 6$ <i>1 mark: 36</i> <i>1 mark: 6 cm</i>	2
6	7.1 cm	$a^2 + a^2 = 100$, so $2a^2 = 100$ and $a^2 = 50$ $a = \sqrt{50} = 7.071... \approx 7.1$ <i>1 mark: $2a^2 = 100$</i> <i>1 mark: 7.1 cm</i>	2
7	6.52 m	$a^2 = 70.56 - 28.09 = 42.47$ $a = \sqrt{42.47} = 6.516... \approx 6.52$ <i>1 mark: 42.47</i> <i>1 mark: 6.52 m</i>	2
8	(a) 52 cm (b) 20 cm	(a) add: $\sqrt{(400 + 2304)} = \sqrt{2704} = 52$ (b) subtract: $\sqrt{(841 - 441)} = \sqrt{400} = 20$ <i>1 mark each</i>	2
9	12 cm	The height cuts the base in half: 5 cm $h^2 = 169 - 25 = 144$, h = 12 <i>1 mark: 5 cm</i> <i>1 mark: 12 cm</i>	2

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Q	Answer	Working / sample response	Marks
10	P = 40 cm; A = 60 cm ²	Third side: $\sqrt{(289 - 64)} = \sqrt{225} = 15$ $P = 17 + 8 + 15 = 40$ $A = \frac{1}{2} \times 8 \times 15 = 60$ 1 mark: 15 1 mark: 40 cm 1 mark: 60 cm²	3

C Reasoning sample answers and marking criteria

/ 9 marks

Q	Answer	Working / sample response	Marks
1	11.7 cm is longer than the hypotenuse; 8 cm	A shorter side can't be longer than the hypotenuse, so he should have subtracted $a^2 = 100 - 36 = 64$, $a = 8$ cm 1 mark: reason 1 mark: 8 cm	2
2	Add to find the hypotenuse; subtract to find a shorter side	Find the right angle. If the unknown side is opposite it, it is the hypotenuse: add the squares. Otherwise subtract the smaller square from the hypotenuse squared. 1 mark: add rule 1 mark: subtract rule	2
3	False	The third side would be $\sqrt{(49 - 49)} = 0$, so there is no triangle The hypotenuse must be strictly longer than each other side. 1 mark: false 1 mark: reason	2
4	Subtract b ² from both sides	$c^2 - b^2 = a^2 + b^2 - b^2 = a^2$ Check: $13^2 - 12^2 = 169 - 144 = 25 = 5^2$ 1 mark: subtract b² 1 mark: result 1 mark: check	3

D Challenge solutions and marking criteria

/ 11 marks

Q	Answer	Working / sample response	Marks
1	8.7 cm	The height bisects the base: 5 cm $h^2 = 100 - 25 = 75$, $h = \sqrt{75} = 8.660... \approx 8.7$ 1 mark: 5 cm and 75 1 mark: 8.7 cm	2
2	9 cm, 40 cm, 41 cm	Let the shorter side be a and the hypotenuse a + 1: $(a + 1)^2 - a^2 = 81$ $2a + 1 = 81$, so $a = 40$ Sides 9, 40, 41 (check $81 + 1600 = 1681$) 1 mark: difference of squares = 81 1 mark: a = 40 1 mark: all three	3

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Q	Answer	Working / sample response	Marks
3	21 cm	$QS = \sqrt{169 - 144} = 5$ $SR = \sqrt{400 - 144} = 16$ $QR = 5 + 16 = 21 \text{ cm}$ <i>1 mark: QS</i> <i>1 mark: SR</i> <i>1 mark: 21 cm</i>	3
4	(5, 12, 13), (9, 12, 15), (12, 16, 20), (12, 35, 37)	$c^2 - b^2 = 144$, so $(c - b)(c + b) = 144$ Factor pairs of the same parity: 2×72 , 4×36 , 6×24 , 8×18 These give $(b, c) = (35, 37)$, $(16, 20)$, $(9, 15)$, $(5, 13)$ <i>1 mark: method</i> <i>1 mark: at least two triangles</i> <i>1 mark: all four</i>	3