

WA8MMGTW3

Two-dimensional space and structures

Core + Challenge

Pythagoras' Theorem: Finding the Hypotenuse — Marking Key

I can identify the hypotenuse and use Pythagoras' theorem to find its length, exactly or rounded.

For tutors and parents. Award method marks for correct working even if the final answer is wrong.

Marks summary

Section A Fluency: 16 Section B Core: 22 Section C Reasoning: 9 Section D Challenge: 11 **Total: 58**

Fluency: 1 mark each; accept an equivalent unsimplified answer for half credit at tutor's discretion.

A

Fluency *answers and working*

/ 16 marks

Q	Answer	Working
1	z	opposite the right angle
2	e	opposite the right angle
3	k	opposite the right angle
4	u	opposite the right angle
5	25	$9 + 16$
6	10	$\sqrt{(36 + 64)} = \sqrt{100}$
7	13	$\sqrt{(25 + 144)} = \sqrt{169}$
8	17	$\sqrt{(64 + 225)} = \sqrt{289}$
9	15	$\sqrt{(81 + 144)} = \sqrt{225}$
10	25	$\sqrt{(49 + 576)} = \sqrt{625}$
11	29	$\sqrt{(400 + 441)} = \sqrt{841}$
12	2.5	$\sqrt{(2.25 + 4)} = \sqrt{6.25}$
13	3.6	$\sqrt{13} = 3.605... \approx 3.6$
14	8.6	$\sqrt{74} = 8.602... \approx 8.6$
15	5.7	$\sqrt{32} = 5.656... \approx 5.7$
16	9, 12, 15	multiply 3, 4, 5 by 3

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B

Core *answers, working and marks*

/ 22 marks

Q	Answer	Working / sample response	Marks
1	$h = 41 \text{ cm}$	$h^2 = 9^2 + 40^2 = 81 + 1600 = 1681$ $h = \sqrt{1681} = 41$ <i>1 mark: correct substitution</i> <i>1 mark: 41 cm</i>	2
2	$x \approx 12.2 \text{ m}$	$x^2 = 7^2 + 10^2 = 49 + 100 = 149$ $x = \sqrt{149} = 12.206... \approx 12.2$ <i>1 mark: $x^2 = 149$</i> <i>1 mark: 12.2 m</i>	2
3	$y = 4 \text{ cm}$	$y^2 = 2.4^2 + 3.2^2 = 5.76 + 10.24 = 16$ $y = \sqrt{16} = 4$ <i>1 mark: 16</i> <i>1 mark: 4 cm</i>	2
4	1000 m (1 km)	$d^2 = 600^2 + 800^2 = 360\,000 + 640\,000 = 1\,000\,000$ $d = 1000 \text{ m}$ <i>1 mark: right triangle set up</i> <i>1 mark: 1000 m</i>	2
5	(a) and (c)	(a) $100 + 576 = 676 = 26^2$ (yes) (b) $36 + 49 = 85$, but $9^2 = 81$ (no) (c) $225 + 400 = 625 = 25^2$ (yes) <i>1 mark each</i>	3
6	1.3 m	$c^2 = 1.44 + 0.25 = 1.69$ $c = \sqrt{1.69} = 1.3$ <i>1 mark: 1.69</i> <i>1 mark: 1.3 m</i>	2
7	7.03 cm	$c^2 = 12.25 + 37.21 = 49.46$ $c = \sqrt{49.46} = 7.0327... \approx 7.03$ <i>1 mark: 49.46</i> <i>1 mark: 7.03 cm</i>	2
8	4.1 m, exact	$c^2 = 0.81 + 16 = 16.81$ $c = \sqrt{16.81} = 4.1$ exactly ($4.1^2 = 16.81$) <i>1 mark: 4.1 m</i> <i>1 mark: exact, with reason</i>	2
9	5.8 cm	Legs: 5 cm across and 3 cm up $c^2 = 25 + 9 = 34$; $c = \sqrt{34} = 5.830... \approx 5.8$ <i>1 mark: legs 5 and 3</i> <i>1 mark: 5.8 cm</i>	2
10	37 cm; 84 cm; 210 cm^2	$c^2 = 144 + 1225 = 1369$, $c = 37$ $P = 12 + 35 + 37 = 84$ $A = \frac{1}{2} \times 12 \times 35 = 210$ <i>1 mark each</i>	3

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C

Reasoning *sample answers and marking criteria*

/ 9 marks

Q	Answer	Working / sample response	Marks
1	$c^2 = a^2 + b^2$ is bigger than a^2 or b^2	c^2 is a^2 plus a positive amount, so $c^2 > a^2$ and $c > a$ (same for b). <i>1 mark: uses the formula</i> <i>1 mark: conclusion</i>	2
2	He added the sides, not the squares; 7.8 cm	$c^2 = 25 + 36 = 61$, $c = \sqrt{61} = 7.81... \approx 7.8$ cm (Also, 11 cm breaks the triangle inequality: $5 + 6$ must be more than c .) <i>1 mark: error explained</i> <i>1 mark: 7.8 cm</i>	2
3	True	$(2a)^2 + (2b)^2 = 4a^2 + 4b^2 = 4c^2$, so the new hypotenuse is $\sqrt{(4c^2)} = 2c$ e.g. 3, 4, 5 $\rightarrow$ 6, 8, 10 <i>1 mark: true</i> <i>1 mark: justification</i>	2
4	Area on the hypotenuse = sum of the other two areas	Areas: $3^2 = 9$, $4^2 = 16$, $5^2 = 25$ $9 + 16 = 25$ The square on the hypotenuse equals the two smaller squares together. <i>1 mark: areas</i> <i>1 mark: $9 + 16 = 25$</i> <i>1 mark: general statement</i>	3

D

Challenge *solutions and marking criteria*

/ 11 marks

Q	Answer	Working / sample response	Marks
1	2; triangle 24	Hypotenuses: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4} = 2$, $\sqrt{5}$, ... Triangle n has hypotenuse $\sqrt{(n+1)}$ $\sqrt{(n+1)} = 5 \rightarrow n+1 = 25 \rightarrow n = 24$ <i>1 mark: 2</i> <i>1 mark: pattern $\sqrt{(n+1)}$</i> <i>1 mark: 24</i>	3
2	25 cm ² both ways	(i) $\text{side}^2 = 3^2 + 4^2 = 25$, so area = 25 cm ² (ii) $7^2 - 4 \times \frac{1}{2} \times 3 \times 4 = 49 - 24 = 25$ cm ² <i>1 mark: Pythagoras</i> <i>1 mark: subtraction</i> <i>1 mark: 25 cm²</i>	3
3	7, 24, 25 and 15, 20, 25	Need $a^2 + b^2 = 625$ Test $a = 1$ to 17: only $49 + 576$ and $225 + 400$ work Answers: 7, 24, 25 and 15, 20, 25 <i>1 mark: method</i> <i>1 mark each triple</i>	3
4	20 and 21	$a^2 + (a+1)^2 = 841$ Try $a = 20$: $400 + 441 = 841$ <i>1 mark: equation or systematic trial</i> <i>1 mark: 20 and 21</i>	2