

WA8MNAUN2

Understanding number

Core + Challenge

Ordering Rational and Irrational Numbers — Marking Key

I can locate, order and compare rational and irrational numbers on a number line using $<$, $>$, $\leq$ and $\geq$.

For tutors and parents. Award method marks for correct working even if the final answer is wrong.

Marks summary

Section A Fluency: 16 Section B Core: 22 Section C Reasoning: 9 Section D Challenge: 11 **Total: 58**

Fluency: 1 mark each; accept an equivalent unsimplified answer for half credit at tutor's discretion.

A Fluency *answers and working*

/ 16 marks

Q	Answer	Working
1	-2.5	halfway between -3 and -2
2	$\frac{1}{2}$	$C = 0.5$
3	D	$\sqrt{2} \approx 1.41$, between 1 and 1.5
4	F	$\pi \approx 3.14$, between 3 and 3.5
5	5 and 6	$25 < 30 < 36$
6	8 and 9	$64 < 70 < 81$
7	1 and 2	$1 < 3 < 4$
8	7	$\sqrt{50} \approx 7.07$ (49 is very close to 50)
9	9	$\sqrt{90} \approx 9.49$
10	$>$	-3 is to the right of -5
11	$>$	$0.666... > 0.6$
12	$>$	$\pi = 3.14159...$
13	$<$	$-0.3 < -0.25$
14	$<$	$3.9^2 = 15.21 > 15$
15	$<$	$-\sqrt{2} \approx -1.41$ is to the right of -1.5
16	$\frac{7}{4}$	$1.7 < 1.732... < 1.75$

Ordering Rational and Irrational Numbers · WA8MNAUN2

B

Core answers, working and marks

/ 22 marks

Q	Answer	Working / sample response	Marks
1	In order: $-1\frac{3}{4}$, $-\sqrt{2}$, 0.4, $\sqrt{3}$, $\frac{9}{4}$	$-1\frac{3}{4} = -1.75$; $-\sqrt{2} \approx -1.41$; 0.4 ; $\sqrt{3} \approx 1.73$; $\frac{9}{4} = 2.25$ $-\sqrt{2} \approx -1.41$ lies just to the right of -1.5 ; $\sqrt{3} \approx 1.73$ lies between 1.5 and 2 <i>1 mark: negatives placed correctly</i> <i>1 mark: 0.4 and $\frac{9}{4}$</i> <i>1 mark: $\sqrt{3}$ between 1.5 and 2</i>	3
2	$2\frac{4}{5}$, $\sqrt{8}$, $\frac{17}{6}$, 2.9	$2\frac{4}{5} = 2.8$; $\sqrt{8} \approx 2.828$; $\frac{17}{6} = 2.8333\dots$; 2.9 $(2.83)^2 = 8.0089 > 8$, so $\sqrt{8} < 2.83 < \frac{17}{6}$ <i>1 mark: decimals</i> <i>1 mark: $\sqrt{8} < \frac{17}{6}$</i> <i>1 mark: correct order</i>	3
3	Wed, Mon, Thu, Fri, Tue	$-1.25 < -1.2 < -0.9 < 0 < 0.5$ <i>1 mark: $-1\frac{1}{4} = -1.25$ coldest</i> <i>1 mark: full order</i>	2
4	6 and 7; 6.3	$36 < 40 < 49$ $6.3^2 = 39.69$ and $6.35^2 = 40.3225$, so $\sqrt{40}$ is between 6.3 and 6.35 $\rightarrow 6.3$ <i>1 mark: 6 and 7</i> <i>1 mark: 6.3 with squaring</i>	2
5	e.g. (a) $\sqrt{6}$ (b) 1.5	(a) $2.4^2 = 5.76$ and $2.5^2 = 6.25$, so $\sqrt{6}$ (≈ 2.449) works (b) $\sqrt{2} \approx 1.41$ and $\sqrt{3} \approx 1.73$: 1.5, $\frac{8}{5}$, 1.7 ... all work <i>1 mark each</i>	2
6	(a) T (b) F (c) F (d) T	(a) $-4 = -4$, so $\leq$ is true (b) $4.5^2 = 20.25 > 20$, so $\sqrt{20} < 4.5$ (c) $-0.4 < -0.35$ (d) $3.14159\dots < 3.142857\dots$ <i>2 marks: all four</i> <i>1 mark: two or three</i>	2
7	-2, -1, 0, 1, 2, 3	$\sqrt{10} \approx 3.16$, so $x \leq 3$ $x > -2.5$, so $x \geq -2$ (-2.5 itself is not an integer) <i>1 mark: ends -2 and 3</i> <i>1 mark: full list</i>	2
8	Q = 2.6, R = $\sqrt{7}$, P = $\frac{8}{3}$	$\sqrt{7} \approx 2.646$ ($2.64^2 = 6.9696$, $2.65^2 = 7.0225$); $\frac{8}{3} = 2.666\dots$ $2.6 < 2.646 < 2.667$: left to right Q, R, P <i>1 mark: decimals</i> <i>1 mark: all matched</i>	2
9	$-\frac{19}{6}$, $-\sqrt{10}$, $-\pi$, -3.1	$\frac{19}{6} = 3.1666\dots$; $\sqrt{10} \approx 3.162$; $\pi \approx 3.142$ for negatives the biggest size is the smallest number <i>1 mark: decimals</i> <i>1 mark: correct order</i>	2

Ordering Rational and Irrational Numbers · WA8MNAUN2

Q	Answer	Working / sample response	Marks
10	(a) $t \geq -3$ (b) $b < 5$	“at least” includes -3: $\geq$ “less than” does not include 5: $<$ <i>1 mark each</i>	2

C

Reasoning *sample answers and marking criteria*

/ 9 marks

Q	Answer	Working / sample response	Marks
1	$-0.5 < -0.25$	-0.5 is further left (further from 0) on the number line than -0.25 so $-0.5 < -0.25$: for negatives, larger size means smaller number <i>1 mark: correct comparison</i> <i>1 mark: number line reason</i>	2
2	$\sqrt{45} > 6.5$	$36 < 45 < 49$, so $6 < \sqrt{45} < 7$ The halfway point is 6.5, and $6.5^2 = 42.25$ $45 > 42.25$, so $\sqrt{45} > 6.5$: it is closer to 7 <i>1 mark: between 6 and 7</i> <i>1 mark: $6.5^2 = 42.25$</i> <i>1 mark: conclusion</i>	3
3	True	Their average is rational and lies between them e.g. between $\frac{1}{3}$ and $\frac{1}{2}$: $(\frac{1}{3} + \frac{1}{2}) \div 2 = \frac{5}{12}$ <i>1 mark: true</i> <i>1 mark: example/method</i>	2
4	No	$x \leq 3$ includes $x = 3$; $x < 3$ does not for integers, $x < 3$ is the same as $x \leq 2$ <i>1 mark: no</i> <i>1 mark: $x = 3$ difference</i>	2

D

Challenge *solutions and marking criteria*

/ 11 marks

Q	Answer	Working / sample response	Marks
1	32	Square each part: $16 < n < 49$ $n = 17, 18, \dots, 48$: $48 - 17 + 1 = 32$ <i>1 mark: $16 < n < 49$</i> <i>1 mark: 32</i>	2
2	$\frac{7}{10}$, $\sqrt{0.5}$, 0.71 , $\frac{5}{7}$	$\frac{5}{7} = 0.714285\dots$; $\frac{7}{10} = 0.7$ $\sqrt{50} \approx 7.07$ ($7.07^2 = 49.98$), so $\sqrt{0.5} \approx 0.707$ $0.7 < 0.707 < 0.71 < 0.714$ <i>1 mark: $\frac{5}{7}$ as a decimal</i> <i>1 mark: $\sqrt{0.5} \approx 0.707$</i> <i>1 mark: correct order</i>	3

Ordering Rational and Irrational Numbers · WA8MNAUN2

Q	Answer	Working / sample response	Marks
3	14 integers; sum 21	$-\sqrt{30} \approx -5.48$ and $\sqrt{70} \approx 8.37$, so the integers are $-5, -4, \dots, 8$ that is $5 + 1 + 8 = 14$ integers -5 to 5 cancel to 0 ; $6 + 7 + 8 = 21$ <i>1 mark: -5 and 8</i> <i>1 mark: 14</i> <i>1 mark: 21</i>	3
4	$\frac{17}{12}$	Test denominators: $\frac{7}{5} = 1.4$, $\frac{10}{7} = 1.428\dots$, $\frac{11}{8} = 1.375$, $\frac{13}{9} = 1.444\dots$, $\frac{14}{10} = 1.4$, $\frac{16}{11} = 1.4545\dots$, none work for denominators 1 to 11 $\frac{17}{12} = 1.41666\dots$, which is between $1.41421\dots$ and 1.42 Check: $(\frac{17}{12})^2 = \frac{289}{144} \approx 2.007 > 2$ <i>1 mark: systematic testing</i> <i>1 mark: $\frac{17}{12}$</i> <i>1 mark: checks it lies in the interval</i>	3