

WA8MNAUN1

Understanding number

Core + Challenge

Rational and Irrational Numbers — Marking Key

I can define and identify rational and irrational numbers and use correct notation for terminating, recurring and rounded decimals.

For tutors and parents. Award method marks for correct working even if the final answer is wrong.

Marks summary

Section A Fluency: 16 Section B Core: 22 Section C Reasoning: 9 Section D Challenge: 11 **Total: 58**

Fluency: 1 mark each; accept an equivalent unsimplified answer for half credit at tutor's discretion.

A Fluency *answers and working*

/ 16 marks

Q	Answer	Working
1	rational	a fraction of integers
2	rational	$\sqrt{9} = 3 = \frac{3}{1}$
3	irrational	10 is not a perfect square
4	irrational	$\pi = 3.14159\dots$ never ends or repeats
5	rational	$-2.5 = -\frac{5}{2}$
6	rational	$0.444\dots = \frac{4}{9}$ (recurring)
7	$\frac{7}{20}$	$\frac{35}{100} = \frac{7}{20}$
8	$-\frac{6}{5}$	$-\frac{12}{10} = -\frac{6}{5}$ (or $-1\frac{1}{5}$)
9	9	$9 \times 9 = 81$
10	$\frac{2}{3}$	$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$
11	0.375	$3 \div 8 = 0.375$ (terminating)
12	0.666...	$2 \div 3 = 0.666\dots$ (recurring)
13	0.27	third digit 2 < 5, round down
14	3.142	$3.1415\dots \rightarrow 3.142$
15	81	$9^2 = 81, 10^2 = 100$
16	7 and 8	$49 < 50 < 64$

B

Core answers, working and marks

/ 22 marks

Q	Answer	Working / sample response	Marks
1	Integers: $-6, \sqrt{36}, -\sqrt{16}$. Rational (not integers): $\frac{5}{8}, 0.1212\dots, 4.25$. Irrational: $\sqrt{7}, \pi$	$\sqrt{36} = 6$ and $-\sqrt{16} = -4$ are integers $0.1212\dots = \frac{12}{99}$ (recurring) and $4.25 = \frac{17}{4}$ are rational 7 is not a perfect square; π never repeats <i>1 mark: integers correct (including $\sqrt{36}$ and $-\sqrt{16}$)</i> <i>1 mark: rational non-integers correct</i> <i>1 mark: $\sqrt{7}$ and π irrational</i>	3
2	$\frac{4}{5}, \frac{-3}{1}, \frac{9}{4}, -\frac{1}{8}$	$0.8 = \frac{8}{10} = \frac{4}{5}$ $-3 = \frac{-3}{1}$ $2\frac{1}{4} = \frac{9}{4}$ $-0.125 = -\frac{125}{1000} = -\frac{1}{8}$ <i>2 marks: all four</i> <i>1 mark: two or three correct</i>	2
3	Between 7 and 8; irrational	$7^2 = 49 < 50 < 64 = 8^2$ 50 is not a perfect square, so $\sqrt{50}$ is irrational <i>1 mark: 7 and 8</i> <i>1 mark: irrational with reason</i>	2
4	(a) 3.14; 3.1416 (b) π is irrational	$\pi = 3.14159265\dots$ (a) 3.14 and 3.1416 (the 5th digit is 9, round up) (b) π 's decimal never ends or repeats, so any decimal we write is rounded <i>1 mark: 3.14</i> <i>1 mark: 3.1416</i> <i>1 mark: irrational, never ends or repeats</i>	3
5	1.333... dollars; \$1.33	$4 \div 3 = 1.333\dots$ (the 3 repeats) to 2 d.p.: 1.33 <i>1 mark: 1.333...</i> <i>1 mark: \$1.33</i>	2
6	(a) 121, 169, 225 (b) $\sqrt{150}, \sqrt{200}$	$11^2 = 121, 13^2 = 169, 15^2 = 225$ 150 and 200 are not perfect squares <i>1 mark each part</i>	2
7	0.636363...; 0.64 is rounded	$7 \div 11 = 0.636363\dots$ (the block 63 repeats) 0.64 is 0.6363... rounded to 2 d.p., so it is not exact <i>1 mark: 0.6363... with ...</i> <i>1 mark: rounded, with reason</i>	2
8	Neither; $\frac{5}{6} = 0.8333\dots$	$5 \div 6 = 0.8333\dots$: only the 3 repeats, the 8 does not 0.83 is rounded (not exact); 0.8383... repeats the wrong block <i>1 mark: neither, with reason</i> <i>1 mark: 0.8333...</i>	2
9	Any two of $\sqrt{10}, \sqrt{11}, \sqrt{12}, \sqrt{13}, \sqrt{14}, \sqrt{15}$	$3^2 = 9$ and $4^2 = 16$ n can be 10, 11, 12, 13, 14 or 15 (none are perfect squares) <i>1 mark each</i>	2

Rational and Irrational Numbers · WA8MNAUN1

Q	Answer	Working / sample response	Marks
10	Irrational	The decimal never ends, and no block of digits repeats (the gaps keep growing) so it is not terminating or recurring, and cannot be written as a fraction <i>1 mark: irrational</i> <i>1 mark: never ends and never repeats</i>	2

C

Reasoning *sample answers and marking criteria*

/ 9 marks

Q	Answer	Working / sample response	Marks
1	No	$\frac{22}{7} = 3.142857142857...$ (recurring: 142857 repeats), so it is rational $\pi = 3.14159...$ is irrational (never repeats) They differ in the 3rd decimal place: $\frac{22}{7}$ is only an approximation of π <i>1 mark: $\frac{22}{7}$ is recurring/rational</i> <i>1 mark: π irrational</i> <i>1 mark: conclusion — equal when rounded, not equal</i>	3
2	True	Any integer n can be written as $\frac{n}{1}$, e.g. $-7 = \frac{-7}{1}$ <i>1 mark: true</i> <i>1 mark: reason</i>	2
3	Square roots of perfect squares are rational	e.g. $\sqrt{25} = 5$ and $\sqrt{\frac{9}{4}} = \frac{3}{2}$ are rational Only square roots of non-perfect squares (like $\sqrt{2}$, $\sqrt{10}$) are irrational <i>1 mark: counterexample</i> <i>1 mark: corrected statement</i>	2
4	The display is rounded	The calculator can only show about 10 digits, so it rounds $\sqrt{2} = 1.41421356237...$ continues forever without repeating (irrational) <i>1 mark: display rounded/limited</i> <i>1 mark: $\sqrt{2}$ is irrational</i>	2

D

Challenge *solutions and marking criteria*

/ 11 marks

Q	Answer	Working / sample response	Marks
1	43	$\sqrt{n}$ is rational only when n is a perfect square: 1, 4, 9, 16, 25, 36, 49 (7 numbers) $50 - 7 = 43$ <i>1 mark: the 7 perfect squares</i> <i>1 mark: 43</i>	2
2	(a) $\sqrt{2} \times \sqrt{8} = 4$ (b) e.g. $\sqrt{2}$ and $5 - \sqrt{2}$ (sum 5)	(a) $\sqrt{2} \times \sqrt{8} = \sqrt{2} \times 2 \times \sqrt{2} = 2 \times (\sqrt{2} \times \sqrt{2}) = 2 \times 2 = 4 = \frac{4}{1}$ (b) $\sqrt{2} + (5 - \sqrt{2}) = 5$, or $\pi + (-\pi) = 0$, etc. <i>1 mark: $\sqrt{2} \times \sqrt{2} = 2$</i> <i>1 mark: = 4, rational</i> <i>1 mark: valid pair in (b)</i>	3

Rational and Irrational Numbers · WA8MNAUN1

Q	Answer	Working / sample response	Marks
3	(a) 8 (b) 447	The block 142857 has 6 digits; $100 = 6 \times 16 + 4$ (a) 4th digit of the block: 8 (b) block sum $1 + 4 + 2 + 8 + 5 + 7 = 27$; $16 \times 27 + (1 + 4 + 2 + 8) = 432 + 15 = 447$ <i>1 mark: $100 = 6 \times 16 + 4$</i> <i>1 mark: 8</i> <i>1 mark: 447</i>	3
4	4.5 cm	$4^2 = 16$ and $5^2 = 25$, so $4 < \sqrt{20} < 5$ $4.4^2 = 19.36 < 20$ and $4.5^2 = 20.25 > 20$, so $4.4 < \sqrt{20} < 4.5$ $4.45^2 = 19.8025 < 20$, so $\sqrt{20} > 4.45$, which rounds to 4.5 <i>1 mark: between 4.4 and 4.5</i> <i>1 mark: tests 4.45^2</i> <i>1 mark: 4.5 cm</i>	3