

YEAR 7 · MATHEMATICS · NUMBER AND ALGEBRA

WA CURRICULUM (SCSA) · UNDERSTANDING NUMBER

Equivalent Fractions: Related and Unrelated Denominators

Explore and represent equivalent fractions visually and numerically, then use multiplicative reasoning to compare fractions and test whether a statement about them is true.

WA7MNAUN1

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Instructions. Work through Sections A to D in order; Section E is the challenge set. Show all working — marks are given for the equivalent fractions you use, not only for the final answer. Leave answers in simplest form unless the question says otherwise. Calculators are not required.

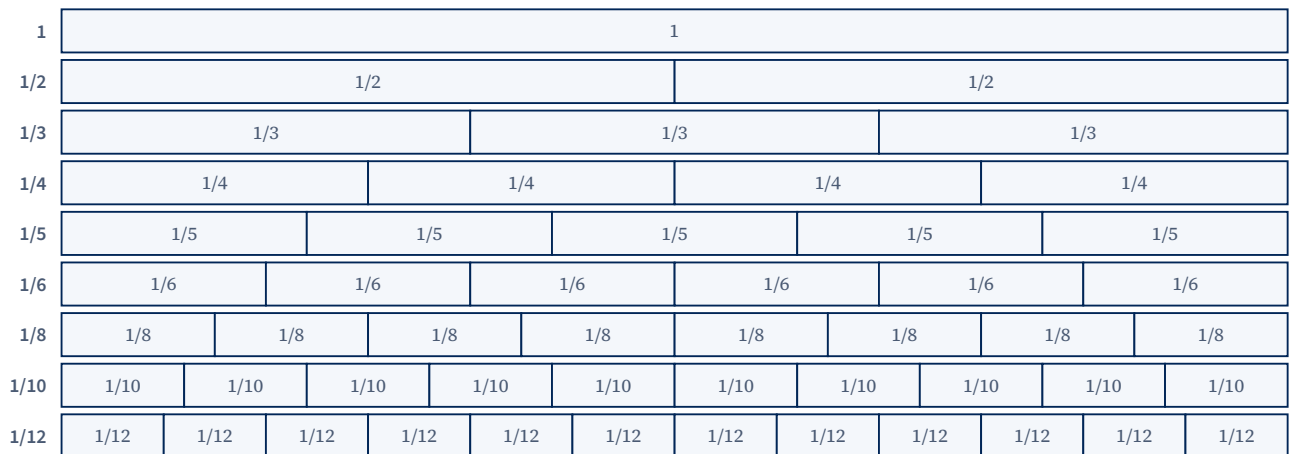
A Seeing equivalence

Use the diagrams. Do not use a calculator.

QUESTIONS 1–5

1 The fraction wall below shows one whole divided in different ways.

4 mk



(a) Write **three** fractions from the wall that are equivalent to $\frac{1}{2}$.

(b) Write **two** fractions from the wall that are equivalent to $\frac{2}{3}$.

(c) Which is larger, $\frac{5}{8}$ or $\frac{3}{4}$? Explain how the wall shows this.

2

3 mk

Three diagrams have been shaded.



Diagram A

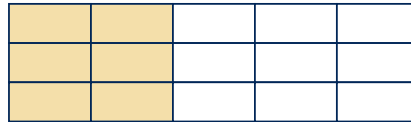


Diagram B



Diagram C

- (a) Write the fraction shaded in each diagram, **using the parts shown** (do not simplify). (b) Which two diagrams show equivalent fractions?

A ____ B ____ C ____

- (c) Explain how you know the remaining diagram is **not** equivalent to the other two.

3

4 mk

The top bar shows $\frac{2}{5}$. In the bottom bar, **each fifth has been split into 3 equal parts**. The thick lines show where the original fifths were.



- (a) The whole is now divided into ____ equal parts. (b) The number of shaded parts is now ____.
- (c) Complete the reasoning:
Each part is ____ times **smaller**, so the number of parts is ____ times **greater**. Therefore $\frac{2}{5} = \frac{\boxed{}}{\boxed{}}$
- (d) Use the same reasoning to complete: $\frac{3}{4} = \frac{\boxed{}}{20}$, because each quarter is split into ____ equal parts.

4

4 mk

The number line below runs from 0 to 1 and is divided into twelfths.



- (a) Mark and label $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{5}{6}$ on the number line. (b) Rewrite each of those four fractions with a denominator of 12.

$$\frac{1}{4} = \frac{\boxed{}}{12} \quad \frac{1}{2} = \frac{\boxed{}}{12} \quad \frac{2}{3} = \frac{\boxed{}}{12} \quad \frac{5}{6} = \frac{\boxed{}}{12}$$

5

3 mk

Shade each diagram so that **all three** show $\frac{3}{4}$.



quarters



eighths



twelfths

Then complete: $\frac{3}{4} = \frac{\boxed{}}{8} = \frac{\boxed{}}{12}$

B

Building equivalent fractions

Show the multiplication or division you used.

QUESTIONS 6–12

6

6 mk

Find the missing value in each pair of equivalent fractions.

(a) $\frac{3}{5} = \frac{\boxed{}}{20}$

(b) $\frac{7}{8} = \frac{21}{\boxed{}}$

(c) $\frac{2}{9} = \frac{\boxed{}}{45}$

(d) $\frac{24}{36} = \frac{\boxed{}}{6}$

(e) $\frac{4}{7} = \frac{20}{\boxed{}}$

(f) $\frac{45}{60} = \frac{3}{\boxed{}}$

7

5 mk

Write each fraction in simplest form.

(a) $\frac{18}{24} = \underline{\hspace{2cm}}$

(b) $\frac{35}{56} = \underline{\hspace{2cm}}$

(c) $\frac{42}{63} = \underline{\hspace{2cm}}$

(d) $\frac{84}{144} = \underline{\hspace{2cm}}$

(e) $\frac{51}{68} = \underline{\hspace{2cm}}$

8

2 mk

Each list is a pattern of equivalent fractions. Write the next three fractions in each list.

(a) $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

(b) $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

9

3 mk

In each row, **circle the fraction that is not equivalent** to the first one.

(a) $\frac{2}{3} \rightarrow \frac{4}{6} \quad \frac{6}{9} \quad \frac{8}{15} \quad \frac{10}{15}$

(b) $\frac{3}{8} \rightarrow \frac{6}{16} \quad \frac{9}{24} \quad \frac{12}{32} \quad \frac{15}{48}$

(c) $\frac{5}{12} \rightarrow \frac{10}{24} \quad \frac{15}{36} \quad \frac{20}{48} \quad \frac{25}{50}$

10

4 mk

Rewrite each pair with a **common denominator**. The denominators are unrelated, so look for the lowest common multiple.

(a) $\frac{3}{4}$ and $\frac{5}{6} \rightarrow \underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$

(b) $\frac{2}{5}$ and $\frac{3}{7} \rightarrow \underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$

(c) $\frac{7}{10}$ and $\frac{11}{15} \rightarrow \underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$

(d) $\frac{5}{8}$ and $\frac{7}{12} \rightarrow \underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$

11

4 mk

These pairs have **unrelated** denominators — you cannot get from one to the other in a single step. Simplify first, then build up.

(a) $\frac{6}{10} = \frac{\boxed{}}{35}$

(b) $\frac{9}{12} = \frac{\boxed{}}{20}$

(c) $\frac{20}{24} = \frac{\boxed{}}{30}$

(d) $\frac{18}{27} = \frac{\boxed{}}{12}$

12

4 mk

True or false? Give a reason for each answer.

(a) $\frac{6}{10} = \frac{9}{15}$ _____

Reason: _____

(b) $\frac{4}{7} = \frac{6}{10}$ _____

Reason: _____

(c) $\frac{12}{18} = \frac{16}{24}$ _____

Reason: _____

(d) $\frac{3}{5} = \frac{5}{7}$ _____

Reason: _____

C

Comparing and ordering

Use equivalent fractions to justify every comparison.

QUESTIONS 13–17

13

5 mk

Insert <, > or = between each pair. Show the equivalent fractions you used.

(a) $\frac{3}{4}$ $\boxed{}$ $\frac{2}{3}$

(b) $\frac{5}{8}$ $\boxed{}$ $\frac{7}{12}$

(c) $\frac{4}{9}$ $\boxed{}$ $\frac{3}{7}$

(d) $\frac{9}{15}$ $\boxed{}$ $\frac{12}{20}$

(e) $\frac{7}{10}$ $\boxed{}$ $\frac{5}{7}$

14

3 mk

Order these fractions from **smallest to largest**.

$\frac{2}{3}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{12}$ $\frac{11}{16}$

Common denominator used: _____

15

4 mk

(a) Which is closer to 1: $\frac{7}{8}$ or $\frac{11}{12}$? Show your working.

(b) Which is closer to $\frac{1}{2}$: $\frac{5}{11}$ or $\frac{7}{15}$? Show your working.

16

4 mk

In Year 7 at Riverbend College, 18 of the 24 students in 7A walk to school, and 21 of the 28 students in 7B walk to school.

(a) Write each as a fraction in simplest form.

7A _____ 7B _____

(b) Does the same fraction of each class walk to school? Explain.

(c) In 7C, 15 of the 25 students walk. Does 7C have a larger or smaller fraction of walkers than 7A?

Justify using equivalent fractions.

17

3 mk

Liam ran $\frac{5}{6}$ of a kilometre. Zara ran $\frac{7}{9}$ of a kilometre.

(a) Who ran further? Show your working using a common denominator.

(b) Write the difference between the two distances as a fraction of a kilometre.

D

Reasoning and communicating

Full sentences are expected. Diagrams support an explanation but do not replace it.

QUESTIONS 18–22

18

4 mk

Complete this justification that $\frac{3}{7} = \frac{12}{28}$, using the idea of parts becoming smaller.

“Each seventh has been split into ____ equal parts, so each part is ____ times smaller. Because the parts are smaller, it takes ____ times as many of them to cover the same amount, so the 3 shaded sevenths become ____ shaded twenty-eighths.”

Now write your own justification, in the same style, for $\frac{2}{5} = \frac{6}{15}$.

19

4 mk

Marcus writes: $\frac{3}{4} + \frac{1}{3} = \frac{4}{7}$. He says “you add the tops and add the bottoms”.

(a) Without doing the addition, explain why $\frac{4}{7}$

cannot be the correct answer. (Hint: compare $\frac{4}{7}$ with $\frac{3}{4}$.)

(b) Use equivalent fractions to work out the correct answer.

(c) Describe Marcus’s mistake in one sentence.

20

4 mk

Ava says: “ $\frac{3}{4}$ is greater than $\frac{2}{3}$ because 3 is greater than 2 and 4 is greater than 3.”

(a) Is Ava’s **answer** correct? Show why using equivalent fractions.

(b) Is Ava’s **reasoning** correct? Find a pair of fractions where her rule gives the wrong answer.

21

4 mk

Multiplying the numerator and denominator by the same number keeps a fraction’s value. Adding the same number to both does not.

(a) Start with $\frac{2}{5}$. Multiply the numerator and denominator by 3. What do you get? Is it equivalent?

(b) Start with $\frac{2}{5}$ again. Add 3 to the numerator and to the denominator. What do you get? Is it equivalent?

(c) Explain why multiplying works but adding does not. Refer to the **size of the parts** in your answer.

22

6 mk

Decide whether each statement is **always**, **sometimes** or **never** true. Justify each answer with an example (and a counter-example where relevant).

- (a) Every fraction can be written in infinitely many equivalent forms.

- (b) If two **positive** fractions have the same numerator, the one with the larger denominator is the smaller fraction.

- (c) Adding 1 to both the numerator and the denominator of a fraction makes it larger.

E

Challenge

Attempt these after completing Sections A to D.

QUESTIONS 23–25

23

4 mk

Find the missing value in each equation.

(a) $\frac{42}{n} = \frac{6}{7}$ $n =$ _____

(b) $\frac{m}{45} = \frac{4}{9}$ $m =$ _____

(c) $\frac{k+2}{15} = \frac{2}{3}$ $k =$ _____

(d) $\frac{3}{x} = \frac{x}{27}$, for positive x $x =$ _____

24

4 mk

Every Year 7 student at a secondary school studies exactly one language: Japanese, Italian or French. Exactly $\frac{3}{8}$ of the cohort chose Japanese, and that came to 45 students.

(a) How many students are in the Year 7 cohort?

(b) $\frac{2}{5}$ of the cohort chose Italian. How many students is that?

(c) The remaining students chose French. What fraction of the cohort chose French, and how many students is that?

25

5 mk

Two jugs of cordial are mixed for a school fundraiser.

Jug A: 2 cups of cordial stirred into 5 cups of water.

Jug B: 3 cups of cordial stirred into 7 cups of water.

(a) What fraction of **Jug A** is cordial? What fraction of **Jug B** is cordial? (Careful: the water is not the whole jug.)

(b) Which jug tastes stronger? Justify your answer using a common denominator.

(c) Both jugs are poured into one large container. What fraction of the mixture is now cordial?

(d) Explain why your answer to (c) must lie between your two answers to (a).

- 1 (a) Any three of $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$. (b) $\frac{4}{6}$, $\frac{8}{12}$. (c) $\frac{3}{4}$ is larger. On the wall the 3-quarter bar reaches further than the 5-eighth bar; $\frac{3}{4} = \frac{6}{8}$ and 6 eighths > 5 eighths.
- 2 (a) $A = \frac{2}{5}$, $B = \frac{6}{15}$, $C = \frac{3}{10}$. (b) A and B, since $\frac{6}{15} = \frac{2}{5}$ (divide both by 3). (c) C is $\frac{3}{10}$; $\frac{2}{5} = \frac{4}{10}$, and 3 tenths $\neq$ 4 tenths, so C is smaller.
- 3 (a) 15. (b) 6. (c) 3 times smaller; 3 times greater; $\frac{2}{5} = \frac{6}{15}$. (d) $\frac{3}{4} = \frac{15}{20}$, each quarter split into 5 equal parts.
- 4 (a) Mark each at its twelfths position: $\frac{1}{4}$ at $\frac{3}{12}$, $\frac{1}{2}$ at $\frac{6}{12}$, $\frac{2}{3}$ at $\frac{8}{12}$, $\frac{5}{6}$ at $\frac{10}{12}$. (b) $\frac{3}{12}$, $\frac{6}{12}$, $\frac{8}{12}$, $\frac{10}{12}$.
- 5 Shade 3 of 4, 6 of 8, 9 of 12. $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$.
- 6 (a) 12 (b) 24 (c) 10 (d) 4 (e) 35 (f) 4
- 7 (a) $\frac{3}{4}$ ($\div 6$) (b) $\frac{5}{8}$ ($\div 7$) (c) $\frac{2}{3}$ ($\div 21$) (d) $\frac{7}{12}$ ($\div 12$) (e) $\frac{3}{4}$ ($\div 17$)
- 8 (a) $\frac{4}{12}$, $\frac{5}{15}$, $\frac{6}{18}$ (b) $\frac{12}{16}$, $\frac{15}{20}$, $\frac{18}{24}$
- 9 (a) $\frac{8}{15}$ — it should be $\frac{8}{12}$. (b) $\frac{15}{48}$ — it should be $\frac{15}{40}$. (c) $\frac{25}{50}$ — it should be $\frac{25}{60}$.
- 10 (a) $\frac{9}{12}$ and $\frac{10}{12}$ (b) $\frac{14}{35}$ and $\frac{15}{35}$ (c) $\frac{21}{30}$ and $\frac{22}{30}$ (d) $\frac{15}{24}$ and $\frac{14}{24}$
- 11 (a) 21 (simplify to $\frac{3}{5}$, then $\times 7$) (b) 15 ($\frac{3}{4}$, then $\times 5$) (c) 25 ($\frac{5}{6}$, then $\times 5$) (d) 8 ($\frac{2}{3}$, then $\times 4$)
- 12 (a) True — both simplify to $\frac{3}{5}$. (b) False — $\frac{4}{7} = \frac{40}{70}$ and $\frac{6}{10} = \frac{42}{70}$. (c) True — both simplify to $\frac{2}{3}$. (d) False — adding 2 to both parts changes the value; $\frac{3}{5} = \frac{21}{35}$ and $\frac{5}{7} = \frac{25}{35}$.
- 13 (a) $>$ ($\frac{9}{12} > \frac{8}{12}$) (b) $>$ ($\frac{15}{24} > \frac{14}{24}$) (c) $>$ ($\frac{28}{63} > \frac{27}{63}$) (d) $=$ (both $\frac{3}{5}$) (e) $<$ ($\frac{49}{70} < \frac{50}{70}$)
- 14 Common denominator 48 (accept any common multiple of 3, 4, 8, 12 and 16): $\frac{7}{12} = \frac{28}{48}$, $\frac{5}{8} = \frac{30}{48}$, $\frac{2}{3} = \frac{32}{48}$, $\frac{11}{16} = \frac{33}{48}$, $\frac{3}{4} = \frac{36}{48}$. Order: $\frac{7}{12}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{11}{16}$, $\frac{3}{4}$.
- 15 (a) $\frac{11}{12}$. Gaps to 1 are $\frac{1}{8} = \frac{3}{24}$ and $\frac{1}{12} = \frac{2}{24}$; the smaller gap wins. (b) $\frac{7}{15}$. Gaps to $\frac{1}{2}$ are $\frac{1}{22}$ and $\frac{1}{30}$; $\frac{1}{30}$ is smaller.

- 16** (a) $7A = \frac{18}{24} = \frac{3}{4}$; $7B = \frac{21}{28} = \frac{3}{4}$. (b) Yes — both simplify to $\frac{3}{4}$, so three quarters of each class walks. (c) Smaller. $7C = \frac{15}{25} = \frac{3}{5}$; with denominator 20, $\frac{3}{4} = \frac{15}{20}$ and $\frac{3}{5} = \frac{12}{20}$, so 7C is smaller.
- 17** (a) Liam. $\frac{5}{6} = \frac{15}{18}$ and $\frac{7}{9} = \frac{14}{18}$, so Liam ran further. (b) $\frac{1}{18}$ of a kilometre.
- 18** Blanks: 4, 4, 4, 12. Sample response for $\frac{2}{5} = \frac{6}{15}$: “Each fifth is split into 3 equal parts, so each part is 3 times smaller. Because the parts are 3 times smaller, 3 times as many are needed to cover the same amount, so 2 fifths becomes 6 fifteenths.”
- 19** (a) $\frac{4}{7}$ is less than $\frac{3}{4}$ ($\frac{16}{28}$ vs $\frac{21}{28}$), but adding a positive fraction must make the total larger than $\frac{3}{4}$ — so the answer is impossible. (b) $\frac{9}{12} + \frac{4}{12} = \frac{13}{12} = 1\frac{1}{12}$. (c) He added numerators and denominators, which treats the parts as if they were the same size; fractions can only be added once they are expressed in equal-sized parts.
- 20** (a) Yes — $\frac{3}{4} = \frac{9}{12}$ and $\frac{2}{3} = \frac{8}{12}$, so $\frac{3}{4}$ is greater. (b) No — *many counter-examples possible*. One is: compare $\frac{2}{3}$ with $\frac{3}{5}$. Here $3 > 2$ and $5 > 3$, so Ava’s rule predicts $\frac{3}{5}$ is larger, but $\frac{2}{3} = \frac{10}{15}$ and $\frac{3}{5} = \frac{9}{15}$, so $\frac{2}{3}$ is actually larger.
- 21** (a) $\frac{6}{15}$ — equivalent. (b) $\frac{5}{8}$ — not equivalent: over a denominator of 40, $\frac{2}{5} = \frac{16}{40}$ but $\frac{5}{8} = \frac{25}{40}$. (c) Multiplying by 3 splits every part into 3 smaller parts and gives 3 times as many of them, so the amount shaded is unchanged. Adding 3 makes 3 extra parts of a **different** size and also changes the size of every part, so the relationship between the shaded amount and the whole changes.
- 22** (a) **Always** — multiply the numerator and denominator by 2, 3, 4, ... without end. (b) **Always** (for positive fractions) — the same number of parts, but each part is smaller; e.g. $\frac{3}{8} < \frac{3}{5}$. (c) **Sometimes** — true for a proper fraction ($\frac{2}{5} \rightarrow \frac{3}{6} = \frac{1}{2}$, larger); false for an improper fraction ($\frac{5}{2} = 2.5 \rightarrow \frac{6}{3} = 2$, smaller); no change when numerator and denominator are equal ($\frac{3}{3} \rightarrow \frac{4}{4}$, both 1).
- 23** (a) $n = 49$ (b) $m = 20$ (c) $k = 8$ (since $k + 2 = 10$) (d) $x = 9$ ($x^2 = 81$)
- 24** (a) $\frac{3}{8}$ of the cohort is 45, so $\frac{1}{8}$ is 15 and the cohort is $8 \times 15 = \mathbf{120}$ students. (Equivalently $\frac{3}{8} = \frac{45}{120}$.) (b) $\frac{2}{5} = \frac{48}{120}$, so **48** students chose Italian. (c) Over a denominator of 40: Japanese $\frac{3}{8} = \frac{15}{40}$ and Italian $\frac{2}{5} = \frac{16}{40}$, together $\frac{31}{40}$. French = $\frac{40}{40} - \frac{31}{40} = \frac{9}{40}$, which is $\frac{9}{40}$ of 120 = **27** students. *Check: $45 + 48 + 27 = 120$.*

- 25** (a) Jug A holds $2 + 5 = 7$ cups, so cordial is $\frac{2}{7}$. Jug B holds $3 + 7 = 10$ cups, so cordial is $\frac{3}{10}$. *Common error: writing $\frac{2}{5}$ and $\frac{3}{7}$, which compare cordial to water rather than to the whole jug.* (b) **Jug B**, but only just: over a denominator of 70, $\frac{2}{7} = \frac{20}{70}$ and $\frac{3}{10} = \frac{21}{70}$. (c) Together there are $2 + 3 = 5$ cups of cordial in $7 + 10 = 17$ cups of mixture, so $\frac{5}{17}$. (d) Over a denominator of 1190: $\frac{2}{7} = \frac{340}{1190}$, $\frac{5}{17} = \frac{350}{1190}$, $\frac{3}{10} = \frac{357}{1190}$ — so $\frac{5}{17}$ sits between them. Mixing a weaker batch with a stronger one cannot give a result outside that range. *Teaching note: part (c) is **not** $\frac{2}{7} + \frac{3}{10}$. Compare with Q19 — when you add fractions the whole stays the same, so adding denominators is wrong; when you pour two jugs together the whole itself grows, so both the cordial and the total are combined.*

WA7MNAUN1 — Explore and represent equivalent fractions with related and unrelated denominators, visually and numerically. Elaborations: using multiplicative relationships to justify equivalence; using equivalence to test the truth of statements about fractions.

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